REVIEW

of

Dissertation for acquiring of scientific and educational degree "Philosophy Doctor"

One candidate: Silvy-Maria Todorova Gurova

Topic of the dissertation: Stochastic Numerical Methods for Estimate of Eigen values

Procedure Notifier: Institute of Information and Communication Technologies – Bulgarian Academy of Sciences, Sofia

Field of higher education: 4. Natural Sciences, Mathematics, and Informatics

Professional direction: 4.5. Mathematics

Doctoral Program: Mathematical Modeling and Application of Mathematics

Opponent: Prof. Michail Todorov, PhD, Section of Differential Equations and Mathematical Physics, Institute of Mathematics and Informatics – BAS, by 101/29.04.2026 of the Director of Institute of Information and Computational Technologies - BAS.

Short biographical record of the applicant

Sylvi-Maria Gurova was born in 1995 in Sofia. In 2013 she graduated from secondary education - SMG. In the period 2013-2019 she studied at the Faculty of Mathematics and Computer Science-SU and graduated successively as a bachelor and master in Applied Mathematics. In parallel with her studies, she worked at the Institute of Information and Communication Technologies-BAS successively as a programmer (2013-2017), mathematician (2017-2019), assistant professor (2019-2023) and from 2023 until now again as a mathematician. In the period 2015-2025 the doctoral candidate is a part-time lecturer at the Faculty of Mathematics and Computer Science-SU, where she leads seminars on stochastic numerical methods and simulations, probability and statistics, differential and integral calculus, linear optimization, etc. In 2020, she was enrolled in full-time doctoral studies at IICT-BAS, from which she was discharged with the right to defend her thesis in 2023.

The presented dissertation is written in Bulgarian and has a volume of 117 pages, B5+1/2 format, including an introduction, 3 chapters and a conclusion, 16 tables, 15 figures and a bibliography of 117 titles, almost all in English.

1. General characterization of the dissertation problem

The problem of finding the spectrum of square symmetric matrices and in particular estimating their extreme eigenvalues occupies a central place in linear algebra due to its key role in a wide range of problems of a scientific and applied nature. Such problems arise in quantum mechanics, physics, financial mathematics, machine learning, signal processing. In tensor analysis and multidimensional data processing, the maximum eigenvalue and spectral norms determine the behavior of iterative methods and regularization techniques. Several numerical approaches to calculating the spectrum have been developed - direct, iterative and stochastic. Direct methods include QR decomposition and solving the characteristic polynomial for matrices with low dimensionality and computational complexity of the order of $O(n^3)$, where n is the dimension of the matrix. A major drawback of exact methods is that for matrices with high dimensionality, calculations with finite accuracy lead to accumulation of errors and hence an incorrect numerical prediction of the eigenvalues. Iterative methods have a lower computational complexity $O(n^2k)$, (n is the matrix dimension, k is the number of iterations), but are slow to converge, have significant memory requirements, and limited parallelization efficiency. Stochastic numerical methods offer an effective alternative, providing a linear relationship between the dimensionality of the problem and the required memory, as well as natural parallelism and relative ease of implementation. Although with slower convergence and lower accuracy compared to deterministic methods, they allow solving problems with very high dimensions and are particularly suitable for estimating extreme eigenvalues of symmetric matrices. The modern development of high-performance computing systems and the development of new hardware architectures implies the search and finding of software solutions based on efficient stochastic algorithms for high-dimensional problems, including for estimating extreme eigenvalues.

The combination of all these activities requires equally good knowledge of the relevant mathematical tools for analytical, computational and software implementation of the relevant models and statements. The topic has a clear foundation and a need for specific applications, which is sufficient justification and motivation for conducting the research. All this assumes the necessary mathematical qualification and knowledge, which the doctoral candidate undoubtedly possesses and skillfully applies.

2. Current state of the problem

The problem of finding the spectrum of matrices usually arises when solving systems of linear algebraic equations (LAEs) and stochastic differential equations (SDEs). In practice, for matrices of large dimensions, the problem is reduced to calculating or at least estimating only the extreme eigenvalues, which determine the condition number and key properties of the system. Such problems arise in image and signal processing, control systems, machine learning, and quantum mechanics, etc.

The first publications related to Monte Carlo (MC) methods in linear algebra date back to the 1950s and 1960s. At that time, deterministic methods such as the Power Method, Rayleigh Coefficient Iteration, the Resolvent Power Method, and the Lanczos algorithm were widely used. The Monte Carlo method for estimating the dominant eigenvalue was proposed by Sobol in 1973, and for the smallest eigenvalue by Mikhailov in 1987. A significant contribution to the development of the Power Monte Carlo (MC) method was made by Dimov and Karaivanova in the period 1996–2000. In the following decades, iterative MC and Quasi-Monte Carlo (QMC) algorithms for finding extreme eigenvalues based on the Power and Resolvent Power Methods were developed. The use of the resolvent matrix for estimating the smallest eigenvalue was introduced by Dimov and Karaivanova in 1998. The first Quasi-Monte Carlo methods for finding extreme eigenvalues were proposed by Karaivanova and Mascani in the period 2001–2003. A number of studies have used QMC methods based on small-discrepancy series, including Sobol, Holton and Fore series, which provide a more uniform and deterministic coverage of the sample space. Compared to standard MC approaches, QMC methods are distinguished by better flexibility, applicability to matrices with high dimensions, relative simplicity and high parallel efficiency. Over the past two decades, various strategies for parallelization of Monte Carlo algorithms have been developed and analyzed. The group around Prof. Ivan Dimov also made a significant contribution here. The main idea in these developments is the balancing of stochastic and systematic error. Hybrid Monte Carlo algorithms for matrix calculations have also been developed, including parallel implementations for estimating the largest and smallest eigenvalues of matrices.

The main goal of the dissertation is to propose and investigate stochastic numerical methods for estimating eigenvalues, by developing and analyzing Monte Carlo and randomized Quasi-Monte Carlo algorithms for approximate calculation of extreme eigenvalues of symmetric square matrices.

3. Methods of investigation

The research methodology in the dissertation is based on fundamental scientific results from the following areas:

- Probability theory and mathematical statistics;
- Mathematical analysis;
- Numerical methods, linear and computational algebra, stochastic numerical methods (Monte Carlo and Quasi-Monte Carlo methods);
- Discrete and analytical number theory;
- Algorithms and data structures; program codes written in Matlab, C++ using pseudo-random number generators such as MT and MS; libraries for randomized Sobol and Holton sequences.

Part of the numerical experiments were performed on a high-performance cluster consisting of 12 servers, and the Sobol sequences were obtained using BRODA's Sobol Randomized Sequence Generator.

The main task of estimating eigenvalues of a symmetric matrix is solved using MC and QMC approaches for numerically solving multidimensional problems and an analysis of the characteristic errors for the methods is performed.

4. Opinion of the author contributions

The Introduction discusses the relevance of the topic, provides an overview of the main scientific results in the field, sets the goals and objectives of the dissertation work, and describes the research methodology in detail. Chapters 1, 2, and 3 are of a contributing nature.

Chapter 1 is devoted to the development of effective stochastic power algorithms for finding the maximum eigenvalue of dense symmetric matrices. The problem is defined and the methods used to solve it are considered: these are the deterministic Power Method and the stochastic Power Method in its two variants - Power Monte Carlo (PMC) and Power Quasi-Monte Carlo (PQMC) method. The construction of the random variable using a Markov chain is described in detail, with an emphasis on the use of almost optimal probabilities. Pseudocode of the developed almost optimal PMC and PCMC algorithms is presented, and the conditions for balancing the stochastic and systematic error are determined.

Chapter 2 considers the problem of estimating the minimum eigenvalue of symmetric square matrices using their resolvent matrices. The Resolvent Power Method and its stochastic implementations using the Resolvent Monte Carlo (RMC) and Resolvent Quasi-Monte Carlo (RQMC) approaches are presented. The chapter investigates the role of parameters affecting the convergence of the infinite series by which the resolvent matrix is represented. Near-optimal RMC and RCMC algorithms are constructed and the balance of stochastic and systematic error is investigated. This balance depends on the number of realizations of the random variables, the length of the Markov chain, the degree of the resolvent matrix, and the accelerating parameter included in the representation of the resolvent matrix as a power series. Numerical experiments are conducted to estimate the minimum eigenvalue of test symmetric matrices.

Chapter 3 is of an applied nature. It applies algorithms from Chapter 1 to a real-world problem in financial mathematics, related to the assessment of the market risk of an investment portfolio. Its assessment is performed using the maximum eigenvalue of the portfolio correlation matrix. Finding the largest eigenvalue is performed using the nearly optimal SMC and SQMC algorithms. When applying the nearly optimal SQMC algorithm, two options are considered: with shuffled Sobol and Holton rows (1) with the "default" setting; and with the use of skip and leap parameters. Numerical experiments with real financial data have been conducted, which demonstrate the applicability and effectiveness of the proposed algorithms.

The main contributions can be divided into two groups:

1. Scientific contributions:

• The Power (Resolvent) Monte Carlo method and the randomized Power (Resolvent) Quasi-Monte Carlo method for estimating extreme eigenvalues of symmetric matrices have been substantiated and studied. Theoretical estimates have been obtained for the dependence between the degree of the resolvent matrix and the length of the Markov chain depending on the accelerating parameter and the norm of the matrix;

• Almost optimal algorithms have been developed, for which the computational complexity and the conditions for the balance between stochastic and systematic error have been estimated. It has been argued that the proposed algorithms lead to a reduction in the variance compared to classical analogues;

2. Scientific and applied contributions:

• Numerical experiments for estimating the maximum eigenvalue of dense symmetric matrices with high dimensions demonstrate the superiority of almost optimal algorithms over their classical analogues both in accuracy and computational complexity;

• It has been numerically shown that the SCMK algorithms require 2-3 more steps in the Markov chain compared to the SMC algorithms. The choice of a pseudorandom number generator or a shuffled series with a small discrepancy has a measurable effect on the accuracy and robustness. In the considered experiments, MT is preferred over MS, and Sobol series over Holton series;

• Numerical experiments for estimating the minimum eigenvalue show that the balance between stochastic and systematic error depends on the number of transitions in the Markov chain, the number of realizations of the random variables $\theta(k)$ and $\theta(k\text{-}1)$, the degree of the resolvent matrix and the acceleration parameter. The results obtained with a randomized Sobol series and with an MT generator for estimating the minimum eigenvalue of symmetric matrices demonstrate the same order;

• Numerical experiments have been conducted for estimating the maximum eigenvalue of two correlation matrices, which is an indicator of market risk concentration. The evaluation was performed using the nearly optimal SMC algorithm with MT/MS pseudorandom number generators and the nearly optimal SQMC algorithm, both without additional settings (default option) and using the skip and leap parameters for shuffled Sobol and Holton series.

• A practical task from the field of financial mathematics is proposed for evaluating the market risk of investment portfolios. The constructed correlation matrix for the respective portfolio is used to evaluate the maximum eigenvalue, which is an indicator of market risk concentration. The evaluation is performed using the nearly optimal Power MC and Power QMC algorithms developed in Chapter 1, and additionally the Power QMC algorithm is modified by using the skip and leap parameters.

5. Critical remarks and recommendations. Literary knowledge

The dissertation makes an excellent impression. It is written in correct Bulgarian. The presentation is concise and logically consistent. I have no critical notes on the substance. The dissertation does not have any proven theorems of its own, but there are well-described algorithms, as well as a detailed comparative analysis and correct citation of the literary sources that are used. Here it is appropriate to note that there is a thorough knowledge of the literature on the issues considered in the dissertation. The literary awareness of the dissertation candidate is based on the latest sources. There is also an excellent symbiosis and complementarity of practical skills and theoretical training.

6. Publications

The results have been reported many times at conferences and seminars in our country and abroad. They have been published in Contemporary Mathematics and Lecture Notes in Computer Science. All works are co-authored with the scientific supervisor. *Contemporary Mathematics* has IF = 2.5 and belongs to Q1, *LNCS* has SJR.

Table: Reference to the works

Articles – 4 pcs	Abroad - 3 pcs. *Cont. Math*, *LNCS*, in Bulgaria 1 pc. NMSCAA'24
Reports at seminars and international scientific events – 5 pcs	MCQMC, LSSC, BGSIAM, AMiTaNS
Participation in projects – 5 pcs.	1 international and 4 national

The doctoral candidate has participated in 5 research projects – one international and 4 national.

From what has been said so far and after consulting the NACID and the Additional requirements of the IICT, it is evident that she meets the minimum requirements for obtaining the ESD "doctor", and according to indicator G she significantly exceeds them. There is no proven plagiarism in the scientific papers submitted for the competition.

7. Importance and contribution to the practice

As I have already noted, there is an excellent symbiosis and complementarity of practical skills and theoretical training. The results obtained in the dissertation have a definite contribution to the SMC and SQMK and financial mathematics. The main goal is to build a stable and computationally efficient methodological framework that provides a manageable balance between systematic and stochastic error and is applicable to high-dimensional problems. Successful implementation and upgrading imply the solution of new interesting problems.

8. Opinion of the abstract of the dissertation

The abstract correctly and completely reflects the content of the dissertation.

9. Personal impression

I have known the doctoral candidate for more than 10 years. She has participated several times with reports and scientific communications at the AMiTaNS conferences. In 2021, she was nominated by *MDPI Axioms* for the best report by a young scientist at the AMiTaNS conference.

Conclusion

Gaining an impression for the doctoral thesis (dissertation) of the applicant and having in mind the legal rules and criteria (LDASRB, Regulations of BAS as well as specific requirements in IICT) I **rate positively** the dissertation. On the strength of virtue of the law **I propose Silvy-Maria Todorova Gurova** for scientific and educational degree Philosophy Doctor (PhD) in Field of Higher Education: 4. Natural Sciences, Mathematics and Informatics, Professional Direction 4.5 Mathematics, Doctoral Program: Mathematical Modeling and Application of Mathematics.

На основание ЗЗЛД

Oppo[illegible]

S[illegible] Equations and
Mathematical Physics, Institute of
Mathematics and Informatics – BAS

Sofia, May 28th 2026